Hawking Radiation as tunneling and the unified first law of thermodynamics for a class of dynamical black holes

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An analysis of relations between the tunneling rate and the unified first law of thermodynamics at the trapping horizons of two kinds of spherically symmetric dynamical black holes is investigated. The first kind is the Vaidya-Bardeen black hole, the tunneling rate $\Gamma \sim e^{\triangle S}$ can be obtained naturally from the unified first law at the apparent horizon, which holds the form $dE_H = TdS + WdV$. Another is the McVittie solution, the action of the radial null geodesic of the outgoing particles does not always has a pole at the apparent horizon, while the ingoing mode always has one. The solution of the ingoing mode of the radiation can be mathematically reduced to the case in the FRW universe smoothly. However as a black hole, the physical meaning is unclear and even puzzling.

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1. Introduction

To establish thermodynamics of dynamical spacetimes and to know how it is related with gravity are important problems in General Relativity. Understanding Hawking radiation is one of the key issues in steps toward this aim. Since Hawking's original work [1], several

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derivations of Hawking radiation have been proposed in the literatures [2]. Recently, a semi-classical tunneling one [3–5], attracts many people's attention. The main ingredient of this method is the consideration of energy conversion in tunneling of a thin sell from the hole. Many works [6] have been investigated for further development of this approach, and the method worked perfectly. However, for criticism and counter criticism see [7]. More recently, general analysis [8–11] using this method gave an interesting result: the tunneling rate $\Gamma \sim e^{\triangle S}$ arises as a consequence of the first law of thermodynamics for horizons holds the form, $TdS = dE_H + PdV$.

However, most investigations of Hawking radiation were based on stationary black hole spacetimes, where the globally defined surface gravity corresponds to the Hawking temperature. Locally, it is not clear whether there is an event horizon associated with a certain dynamical spacetime and this causes the difficulty to discuss Hawking radiation for a dynamical situation. Recently, Hayward *et al.* [12] proposed a locally defined Hawking temperature for dynamical black holes where the Parikh-Wilczek [4] tunneling method was used. Further, using the tunneling method, there has been proved to be a Hawking radiation associated with the locally defined apparent horizon of the Friedmann-Robertson-Walker (FRW) universe [13], where the Hawking temperature was measured by an observer with the Kodama vector [14] inside the horizon.

Even in a dynamical spacetime the tunneling method seems powerful! This motivate us to consider: does the result obtained in [8–11] is still effective in dynamical background spacetimes? If it is true, it must cast some lights on the thermodynamics of dynamical spacetimes. In a previous paper [15], we have successfully extended the result to the case of the FRW universe. Based on the unified first law $dE_H = TdS + WdV$ holding on the apparent horizon, we obtained the tunneling rate naturally. The dynamical surface gravity still linked to a Hawking temperature, which was measured by a Kodama vector. However, for dynamical black holes, it does not seem obviously correct. In this letter, we would like to investigate the case for two kinds of dynamical black holes. The first is the black holes in the Vaidya spacetime [16], which have been vastly discussed in literatures. Another is the McVittie spacetime, which is just the Schwarzschild black hole embedded in a dynamical

background, the FRW universe. It has often been used to study the effect of the universe's expansion on solar system dynamics [17, 18]. Both of the solutions are not precisely the standard notions of black holes, but they still have horizons which the familiar black hole theorems seem to holds. Based on the above two cases, Hawking radiation as tunneling from the trapping horizons using the Hamilton-Jacobi method [5] were analyzed in [19].

We present our analysis of the two kinds of dynamical back holes in Sec. 2 and in Sec. 3, respectively. In the last section we give out our conclusions and a brief remark.

In this Letter we take the unit convention $G = c = k = \hbar = 1$.

2. The Vaidya-Bardeen black hole

Let us consider a spherically symmetric spacetime, which is a typical dynamical one, the Vaidya black hole. The metric of the 4-dimensional Vaidya spacetime can be written as

$$ds^2 = -e^{2\Psi(r,v)}A(r,v)dv^2 + 2e^{\Phi(r,v)}dvdr + r^2 d\Omega^2, \tag{1}$$

where $A(r,v) = 1 - 2m(r,v)/r$, r is the radius coordinate, v is an advanced null coordinate, and $d\Omega^2$ is the line element of a two-dimensional unit sphere. Following [19], for the special case $\Psi(r,v) = \Phi(r,v)$, we call it the Vaidya-Bardeen metric. The metric (1) can be rewritten as $ds^2 = h_{ab}dx^a dx^b + r^2 d\Omega^2$, with $x^a = (v,r)$.

For dynamical black holes, we prefer to Kodama-Hayward (K-H) theory [20, 21], where two conserved currents can be introduced in spherical dynamical systems. The first is the Kodama vector $K^a = -\epsilon^{ab}\nabla_b r$, ϵ^{ab} denotes the volume form. For the metric (1) we have $K^a = e^{-\psi(v,r)}(\partial_v)^a$, and the corresponding conserved charge is the three dimensional volume $V = \int_\sigma K^a d\sigma_a = 4\pi r^3/3$, where $d\sigma_a$ is the volume form times a future directed unit normal vector of the space-like hypersurface σ_a. Another is defined as the energy-momentum density $j^a = T^a_b K^b$ along the Kodama vector, and the conserved charge is $E = -\int_\sigma j^a d\sigma_a$, which is equal to the Misner-Sharp energy. The apparent/trapping horizon[27] is defined by $h^{ab}\partial_a r \partial_b r = 0$. So we have $A(r,v) = 0$, which leads to the horizon $r_H = r_H(v) = 2m(v, r_H(v))$. According

to the definition, the Misner-Sharp energy inside the apparent horizon $r = r_H$ is

$$E_H = \frac{r}{2}(1 - h^{ab}\partial_a r \partial_b r)|_{r=r_H} = \frac{r_H}{2}. \tag{2}$$

The surface gravity associated with the Vaidya-Bardeen dynamical horizon takes

$$\kappa = \frac{1}{2}\nabla^a \nabla_a r|_{r=r_H} = \frac{A'(r,v)}{2}|_{r=r_H} = \frac{1}{2r_H} - \frac{m'(r_H,v)}{r_H}, \tag{3}$$

where the prime denotes the derivative with respect to r.

The unified first law of thermodynamics at the apparent horizon in theVaidya spacetime holds the form [22, 23]

$$dE_H = TdS + WdV, \tag{4}$$

where $W = -1/2h^{ab}T_{ab}$ is the work term. For the metric (1), by Einstein's equations $G^a_b = 8\pi T^a_b$, one can find $T^v_v = T^r_r = -1/(4\pi r_H^2)\partial m/\partial r|_{r=r_H}$, so we have

$$W = -\frac{1}{2}(T^v_v + T^r_r) = -T^v_v = -\frac{1}{4\pi r_H^2}\frac{\partial m}{\partial r}|_{r=r_H}. \tag{5}$$

The identity (4) can be understood from two different sides. In standard thermodynamics, it is a connection between two quasi-static equilibrium states of a system, which differing infinitesimally in the extensive variables volume, entropy, energy by dV, dS and dE_H, respectively, while having the same values the intensive variables temperature T, and work density W. Both of the two states are spherically symmetric solutions of Einstein equations with the radius of horizon differing by dr_H while having the same source $T_{\mu\nu}$. Dynamically, it is the energy balance under infinitesimal virtual displacements of the horizon normal to itself. From this perspective, the identity (4) must be linked with conservation of energy and thus to the tunneling process.

Corresponding the above two understanding, the whole setup can be considered from two different sides. First, as a result of tunneling, some matter either tunnels out or in across the horizon, therefore the energy of the whole spacetime changes, thus the energy attributed to the shell should be given out. Second, considering the s-wave WKB approximation, the imaginary part of the action is directly related with the Hamiltonian of tunneling particles. Thus, the first law of thermodynamics is crucial to connect the above two sides, energy changes of whole spacetime and the Hamiltonian of tunneling particles.

The radial null geodesic ($ds^2 = d\Omega^2 = 0$) near the apparent horizon for the metric (1) is

$$\dot{r} \equiv \frac{dr}{dv} = \frac{1}{2}A(r,v)e^{\psi(r,v)} \simeq e^{\psi(r_H,v)}\kappa(r - r_H), \tag{6}$$

where κ is the surface gravity (3).

The imaginary part of the action for an s-wave outgoing positive energy particle which crosses the horizon outwards from r_i to r_f can be expressed as

$$\begin{aligned} \mathrm{Im}\mathcal{S} &= \mathrm{Im}\int_{r_i}^{r_f} dr = \mathrm{Im}\int_{r_i}^{r_f}\int_{0}^{p_r} dp_r' dr \\ &= \mathrm{Im}\int_{\mathcal{H}_i}^{\mathcal{H}_f}\int_{r_i}^{r_f} \frac{dr}{\dot{r}} d\mathcal{H} = -\int_{\mathcal{H}_i}^{\mathcal{H}_f} \frac{d\mathcal{H}}{2T} e^{-\psi(r_H,v)}, \end{aligned} \tag{7}$$

where we have used the Hamilton's equation $\dot{r} = d\mathcal{H}/dp_r|_r$, the relation between Hawking temperature and surface gravity, $T = \kappa/2\pi$, and a contour integral at the pole $r = r_H$. Evaluating of the integral (7), the form of the Hamiltonian $d\mathcal{H}$ is necessary to be determined out. For this, we turn to the system, appealing to energy conversation, to guess the form of $d\mathcal{H}$. Since the system is explicit time dependence , the Hamiltonian is no-longer equal to the total energy of the system. Luckily, according to K-H theory, we still can determine out the relation between the Hamiltonian and the total energy of the system.

The total energy of the spacetime can be expressed as

$$E_T = \frac{r_H}{2} - \int_{\sigma} T_b^a K^b d\sigma_a, \tag{8}$$

where the first term corresponds to the energy (2) inside the apparent horizon, the second term corresponds to the outside and the integration extends from the apparent horizon to infinity. Now, we can give the energy changes between the final and initial states of the tunneling process, which contributes to the shell in the view of a Kodama observer. By energy conservation we have

$$\begin{aligned} d\mathcal{H}_K =& E_T^f(r_H + \delta r_H) - E_T^i(r_H) = \frac{\delta r_H}{2} - \left(\int_{r_H+\delta r_H}^{\infty} - \int_{r_H}^{\infty}\right) T_v^v K^v d\sigma_v \\ =& dE_H + T_v^v dV = dE_H - W dV, \end{aligned} \tag{9}$$

where W is the work term (5).

Since the energy difference $d\mathcal{H}_K$ is measured by a Kodama observer inside the apparent horizon, in our case near the apparent horizon, we have

$$d\mathcal{H} = \frac{d\mathcal{H}_K}{e^{-\psi(v,r)}|_{r=r_H}} = e^{\psi(v,r_H)} d\mathcal{H}_K. \tag{10}$$

Substituting (9) (10) into (7), one can obtain

$$\mathrm{Im}\mathcal{S} = -\int_{\mathcal{H}_i}^{\mathcal{H}_f} \frac{d\mathcal{H}}{2T} = -\int \frac{dE_h - WdV}{2T}. \tag{11}$$

Using the first law of thermodynamics (4) on the apparent horizon, from (11) we finally have

$$\mathrm{Im}\mathcal{S} = -\int \frac{dS}{2}. \tag{12}$$

Now, one can immediately have the semi classical tunneling rate from the Vaidya-Bardeen black hole, $\Gamma \sim e^{-2\mathtt{Im}\mathcal{S}} = e^{\int_{S_i}^{S_f} dS} = e^{+\triangle S}$, with $\triangle S = S_f - S_i$. This is the well-known result obtained in [4] for a general, stationary, asymptotically flat, spherically symmetric background. And as a consequence of the first law of thermodynamics, this result appeared in the discussion of a static, spherically symmetric spacetime in [9, 10]. Here, we have recovered it in a background of dynamical spacetime, the Vaidya-Bardeen black hole.

3. The McVittie solution

In this section, we will analyze another dynamical black holes, the McVittie solution. As we will see below, preferring to the K-H theory, the radiation of this kind of dynamical black hole is puzzling. In 4-dimensional spacetime the metric of the McVittie solution is given by [24]

$$ds^2 = -A(r,t)dt^2 + B(r,t)(dr^2 + r^2 d\Omega^2), \tag{13}$$

where

$$A(r,t) = \left[\frac{1-(\frac{m}{a(t)r})}{1+(\frac{m}{a(t)r})}\right]^2, B(r,t) = a^2(t)\left[1-\left(\frac{m}{a(t)r}\right)\right]^2. \tag{14}$$

When $m = 0$, the metric (13) reduces to a flat FRW solution with the scale factor $a(t)$; while when $a(t) = 1$ it becomes a Schwarzschild metric with mass m. Takeing the so-called Nolan gauge [25], the metric (13) can be expressed as

$$ds^2 = -(A_s - H^2(t)r^2)dt^2 + A_s^{-1}dr^2 - 2A_s^{-1/2}H(t)rdrdt + r^2d\Omega^2, \tag{15}$$

where $r \in (2m, \infty)$, $A_s \equiv 1 - 2m/r$, and $H(t) = \dot{a}/a$ is the Hubble parameter.

The energy momentum tensor of the homogeneous perfect fluid takes

$$T^\mu_\nu = \mathrm{diag}(\rho, -p, -p, -p), \tag{16}$$

where $\rho = \rho(t)$ is the energy density, and $p = p(t)$ is the pressure of the perfect fluid. The Einstein-Friedmann equations read

$$3H^2 = 8\pi\rho, \quad 2A_s^{-1/2}\dot{H} + 3H^2 = -8\pi p. \tag{17}$$

The metric (15) can be rewritten as $ds^2 = h_{ab}dx^a dx^b + r^2 d\Omega^2$, with $x^a = (t, r)$. Here the McVittie black hole is really a fake dynamical one since we let the mass $m = \mathrm{const}$. However, it is still has dynamical horizons.

The apparent/trapping[28] horizon can be given by $h^{ab}\nabla_a r \nabla_b r = 0$, and using the metric (15) we have the relation

$$A_s|_{r_H} \equiv 1 - \frac{2m}{r_H} = H^2(t)r_H^2, \tag{18}$$

which determines the radius of the apparent horizon $r_H = r_H(t)$. The Misner-Sharp mass inside the horizon takes

$$E_H = \frac{r}{2}(1 - h^{ab}\partial_a r \partial_b r)|_{r=r_H} = \frac{r_H}{2} = m + \frac{1}{2}H^2(t)r_H^3. \tag{19}$$

For the metric (15), the surface gravity is $\kappa = \frac{1}{2}\nabla^a\nabla_a r|_{r=r_H} = m/r_H^2 - H^2 r_H^2 - \dot{H}/(2H)$. Using (18) it can be expressed as

$$\kappa = \frac{3m}{r_H^2} - \frac{1}{r_H}\left(1 - \frac{3H^2r_H^2 - 1}{4H^2r_H^2}\dot{r}_H\right). \tag{20}$$

One can easily see when $a(t) = \mathrm{const}$, it is just the surface gravity of the Schwarzschild black hole at the even horizon, which takes $\kappa = 1/(2r_H)$ with $r_H = 2m$. While when $m = 0$, it

reduce to the surface gravity of a flat FRW universe at the apparent horizon $r_H = 1/H(t)$, which is $\kappa = -\left(1 - \dot{r}_H/(2Hr_H)\right)/r_H$ [26].

Since we prefer to the K-H theory, we assume that the unified first law of thermodynamics at the apparent/trapping horizon of the McVittie black hole still takes [19]

$$dE_H = TdS + WdV, \tag{21}$$

with the work term $W = (\rho - p)/2$, and $S = \pi r_H^2$. We interpret S as the entropy inside the apparent/trapping horizon. A more detailed discussion is indeed need, however, it's beyond our present paper.

The radial null geodesic for the metric (15) is

$$\dot{r} \equiv \frac{dr}{dt} = Hr\sqrt{A_s} \pm A_s, \tag{22}$$

where $+/-$ corresponding the outgoing/ingoing positive energy particles respectively. However, from (22) one can see that for the outgoing mode, the action at the apparent horizon $r = r_H$, does not always has a pole, unless the expansion rate is slowly enough ($H = \dot{a}/a \simeq 0$). In that case it reduces to a Schwarzschild one, which the Hawking radiation has been vastly discussed in literatures. While the action of the ingoing mode always has a pole at the horizon. So, from the view of tunneling, the McVittie solution is more of a FRW universe than a Schwarzschild black hole.

In the following, we would like to investigate the tunneling process of the ingoing particles. Using (18) and (20), near the horizon r_H , the equation (22) can be rewritten as

$$\begin{aligned} \dot{r} &\simeq -\left(\frac{3m}{r_H^2} - \frac{1}{r_H}\right)(r - r_H) \\ &= -\left[1 + \frac{3H^2 r_H^2 - 1}{4H^2 r_H^3}\left(\frac{3m}{r_H^2} - \frac{1}{r_H}\right)^{-1} \dot{r}_H\right]^{-1} \kappa(r - r_H). \end{aligned} \tag{23}$$

Investigating an ingoing mode, we hope the McVittie spacetime can be smoothly reduced to the FRW universe. Assuming that the mass m is small enough, so we have $\kappa \simeq -\left(1 - \dot{r}_H/(2Hr_H)\right)/r_H < 0$ [26].

Similar with the procedure in the Sec. 2, the imaginary part of the action for an s-wave ingoing positive energy particle which crosses the horizon inwards from r_i to r_f takes

$$\mathrm{Im}\mathcal{S} = \int_{\mathcal{H}_i}^{\mathcal{H}_f} \frac{d\mathcal{H}}{2T}\left[1 + \frac{3H^2r_H^2 - 1}{4H^2r_H^3}\left(\frac{3m}{r_H^2} - \frac{1}{r_H}\right)^{-1}\dot{r}_H\right], \tag{24}$$

here, the relation between Hawking temperature and surface gravity is $T = |\kappa|/2\pi$.

Corresponding the metric (15), the Kodama vector is $K^a = (\partial_t)^a$. Interestingly, the Kodama vector is accidentally equal to the timelike Killing vector in stationary black hole systems. So, from this point of view, the radiation is more of a stationary Schwarzschild black hole than the dynamical FRW universe. As we have analyzed in Sec. 2, in this situation, the total energy of spacetime still takes $E_T = r_H/2 - \int_\sigma T_b^a K^b d\sigma_a$. The energy contributing to the shell in the view of a Kodama observer is

$$\begin{aligned} d\mathcal{H}_K =& E_T^f(r_H + \delta r_H) - E_T^i(r_H) = \frac{\delta r_H}{2} - \left(\int_{r_H+\delta r_H}^{\infty} - \int_{r_H}^{\infty}\right) T_v^v K^v d\sigma_v \\ =& dE_H - \rho dV. \end{aligned} \tag{25}$$

Since the Kodama vector is equal to the timelike Killing vector, we have $d\mathcal{H} = d\mathcal{H}_K$. From (24) and (25) one obtains

$$\mathrm{Im}\mathcal{S} = \int \frac{dE_H - \rho dV}{2T}\left[1 + \frac{3H^2r_H^2 - 1}{4H^2r_H^3}\left(\frac{3m}{r_H^2} - \frac{1}{r_H}\right)^{-1}\dot{r}_H\right]. \tag{26}$$

The integrated term of the above equation can be further simplified. Using (17) and (19), we get

$$\begin{aligned} dE_H &= H\dot{H}r_H^3 dt + \frac{3}{2}H^2 r_H^2 dr_H = \frac{4\pi}{3}r_H^3\dot{\rho}dt + \frac{3H^2}{8\pi}dV \\ &= Vd\rho + \rho dV = d(\rho V). \end{aligned} \tag{27}$$

Combining (17), (18), and (27), we have

$$\begin{aligned} &(dE_H - \rho dV)\frac{3H^2r_H^2 - 1}{4H^2r_H^3}\left(\frac{3m}{r_H^2} - \frac{1}{r_H}\right)^{-1}\dot{r}_H \\ &= Vd\rho\frac{1}{2H^2r_H^2}\dot{r}_H = -\frac{4}{3}\pi\dot{\rho}\frac{r_H}{2H^2}dr_H = -\frac{\dot{H}r_H}{2H}dr_H \\ &= 2\pi(\rho + p)r_H^2 dr_H = \frac{1}{2}(\rho + p)dV. \end{aligned} \tag{28}$$

Substituting (28) into (26), one have

$$\mathrm{Im}\mathcal{S} = \int \frac{dE_H - WdV}{2T} \tag{29}$$

where $W = (\rho - p)/2$ is the work term. Using the first law of thermodynamics (21) on the horizon r_H, the semi classical tunneling rate takes $\Gamma \sim e^{-2\mathtt{Im}\mathcal{S}} = e^{-\int_{S_i}^{S_f} dS} = e^{-\triangle S}$ with $\triangle S = S_f - S_i$. Indeed, the result can be mathematically reduced to the case of the FRW universe [15] smoothly. However, as a black hole, the physical meaning of the ingoing mode radiation of the McVittie spacetime is puzzling.

4. Conclusion and Remarks

In this Letter preferring to the K-H theory, we have extended the work [8–10] to investigate two kinds of dynamical black holes, the Vaidya-Bardeen black hole and the McVittie black hole. In the Vaidya-Bardeen spacetime, the tunneling rate $\Gamma \sim e^{\triangle S}$ can be obtained naturally from the unified first law at the apparent horizon, which holds the form $dE_H = TdS + WdV$. In the McVittie case, we find the action of the radial null geodesic of the outgoing particles does not always has a pole at the apparent horizon, while the ingoing mode always has one. From the view of tunneling, the McVittie black hole is more of the FRW universe than a Schwarzschild black hole. Assuming the mass m is small enough, the McVittie solution can be reduced to the FRW universe smoothly. The tunneling rate of ingoing particles still can be obtained from the unified first law holds on the apparent horizon, where the procedure mathematically resembles with the case in the FRW universe [15]. However, as a black hole, the physical meaning of this kind radiation is unclear and even puzzling. In this sense, the McVittie spacetime may also not actually be viewed as a dynamical black hole, despite its resemblance.

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[27] In the generalized Vaidya spacetime, the apparent horizon is just a kind of trapping horizon defined in K-H theory. So we don't distinguish the two concepts in our paper. For a more detailed discussion, we prefer the reader to [22].

[28] Here we still not distinguish the two concepts, since in our calculation, using the definition of the apparent horizon, which is equal to the trapping horizon exactly from the K-H theory in [19]